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ON A SIMPLE CORRELATION FOR TOTAL BAND ABSORPTANCE OF RADIATING GASES†

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NOMENCLATURE

A,	total band absorptance [cm^{-1}];
$\bar{A}$,	dimensionless total band absorptance;
C_1, C_2, C_3,	characteristic band parameters [M^2g^{-1}, $Mg^{-\frac{1}{2}}cm^{-1}$, cm^{-1}];
D,	diameter [cm];
f_2,	dimensionless function defined in equation (2);
h,	spacing between two planes [cm];
L,	mean beam length [g cm^{-2}];
p_e,	dimensionless effective broadening pressure;
r,	geometric beam length [cm];
t,	dimensionless broadening parameter;
u,	dimensionless mass pathlength defined in equation (3);
X,	mass pathlength [g cm^{-2}].

Greek symbols

ρ,	density [g cm^{-3}];
ϕ,	angle [deg];
Ω,	solid angle [steradian].

INTRODUCTION

THE USE of total band absorptance for calculating radiant energy transfer in bounded and unbounded systems involving infrared radiating gases has received considerable attention recently. A discussion of the role of total band absorptance and various total band absorptance correlations has been given in a recent review article [1]. In particular, the single continuous correlation proposed by Tien and Lowder [2] on the basis of the Edwards wide-band model [3] has found wide applications in radiative transport analyses [4–6]. This correlation, however, is still complex enough to restrict severely its usefulness in many calculations. For instance, the calculation of mean beam length [7, 8] could not be achieved in a simple fashion based on this correlation (see the section of Concluding Remarks in [2]). The present note is to propose a simpler correlation for applications under certain thermodynamic conditions.

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A SIMPLE TWO-PARAMETER CORRELATION

The correlation of Tien and Lowder for total band absorptance is expressed in dimensionless form as [2, 1]

$$\bar{A}(u, t) = \ln\left\{u f_2(t)\left[\frac{u + 2}{u + 2f_2(t)}\right] + 1\right\} \tag{1}$$

where

$$f_2(t) \equiv 2{\cdot}94\,[1 - \exp(-2{\cdot}60\,t)] \tag{2}$$

and

$$\bar{A} \equiv A/C_3, \qquad u \equiv (C_1/C_3)X, \qquad t \equiv (C_2^2/4C_1C_3)p_e \tag{3}$$

where A is the total band absorptance, X is the mass pathlength, and p_e is the equivalent broadening pressure. The constants C_1, C_2 and C_3 are the characteristic spectroscopic parameters of the Edwards wide-band model and are generally functions of temperature only. For infrared bands of common radiating gases, these temperature-dependent parameters have been tabulated by Edwards *et al.* [9, 1].

One important characteristic of the total band absorptance correlation is that at moderately large pressures, only two parameters (C_1 and C_3) are required for the correlation. This two-parameter representation is approximately valid in the region that t is of the order $0[10^{-1}]$ or higher, where the dependence of A on t is indeed negligible as shown in Fig. 1. It should be noted that this particular region covers a wide range of thermodynamic conditions in actual physical situations. Specifically, for example, according to the tabulated information [9, 1], the 4·3 μ band of CO_2 gives $t = 0{\cdot}190\,(T/100)^{\frac{1}{2}}\,p_e$, and the 2·7 μ band of H_2O gives $t = 0{\cdot}175\,(100/T)^{\frac{1}{2}}\,p_e$, where T is in degrees Kelvin and p_e is in atmospheres. The indicated region of t for a

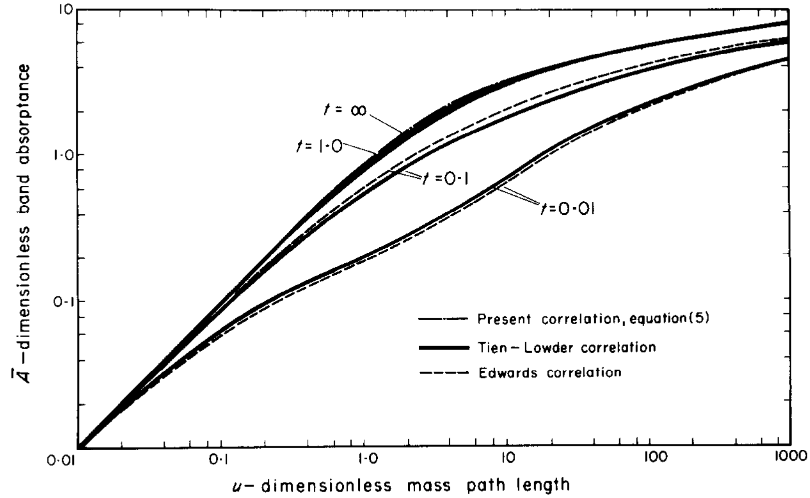

FIG. 1. Comparison among the present correlation, Tien–Lowder correlation and Edwards correlation.

two-parameter characterization of total band absorptance can be easily realized under normal thermodynamic conditions.

On the basis of the above discussion, the two-parameter correlation for t of the order $0[10^{-1}]$ or higher warrants further consideration for possible simplification of the complex analytical form of the correlation. In the limit of large t, equation (1) reduces to

$$\bar{A}(u) = \ln\left\{2{\cdot}94\,u\left[\frac{u+2}{u+5{\cdot}88}\right]+1\right\} \tag{4}$$

which is rather cumbersome for mathematical manipulation. In this limit, on the other hand, a simpler correlation can be established:

$$\bar{A}(u) = \sinh^{-1} u \tag{5}$$

which not only satisfies all the mathematical properties required for a total band absorptance function [7, 2], but also agrees well with the existing correlations as shown in Fig. 1. Throughout the whole range of u, the maximum deviation of equation (5) from equation (4) never exceeds seven percent. The advantage of using equation (5) for the total band absorptance correlation lies primarily in the fact that extensive mathematical relations and formulas concerning inverse hyperbolic functions are available [10]. This will be clearly demonstrated in the following application to the calculation of mean beam length for radiating gases.

MEAN BEAM LENGTH CALCULATIONS

In terms of total band absorptance, the mean beam length L of a gas body is defined through the following relationship [7, 8]:

$$A(L) = \frac{1}{\pi}\int_{\Omega} A(r)\cos\phi\,\mathrm{d}\Omega \tag{6}$$

where r is the distance from the differential element under consideration to the confining surface of the gas body, ϕ is the angle between r and the normal to the differential element, and Ω is the solid angle subtended by the gas body. In the past, mean beam lengths for common gas bodies have been obtained analytically for certain optical pathlength regions characterized by the power-law and logarithmic correlations [8]. But, due to the complex form of existing total band absorptance correlations, no analytical expressions of the mean beam length for the whole range of optical pathlengths have ever been achieved. With the simple form of the correlation given in equation (5), however, this is no longer a problem for simple gas-body geometries. Two simple cases will be considered here for illustration.

For a spherical enclosure of radiating gas, the mean beam length L based on equation (5) can be expressed by way of simple direct integration of equation (6) as

$$\left(\frac{L}{L_0}\right) = \left(\frac{3}{2u_0}\right)\sinh\left\{\frac{1}{2}\left[\left(2+\frac{1}{u_0^2}\right)\sinh^{-1}u_0 - \left(1+\frac{1}{u_0^2}\right)^{\frac{1}{2}}\right]\right\} \tag{7}$$

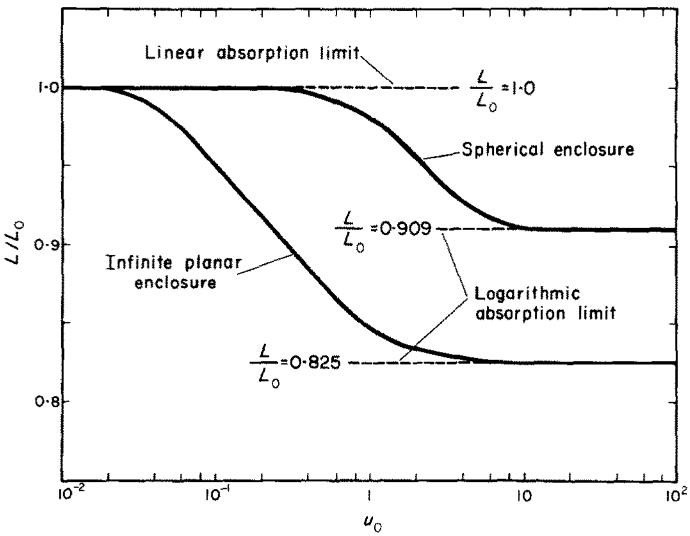

FIG. 2. Mean beam lengths for spherical and infinite planar enclosures.

where L_0 is the geometric mean beam length [7, 8] and for this case $L_0 = 2D/3$, D is the enclosure diameter, $u_0 = (C_1/C_3)\rho D$, and ρ is the density of the radiating gas. For an enclosure bounded by two infinite parallel planes, the corresponding result is

$$\left(\frac{L}{L_0}\right) = \left(\frac{1}{2u_0}\right) \sinh \left\{ \sinh^{-1} u_0 + u_0^2 \left[\left(1 + \frac{1}{u_0^2}\right)^{\frac{1}{2}} - 1 \right] \right\} \quad (8)$$

where $L_0 = 2h$, h is the distance between two planes, and $u_0 = (C_1/C_3)\rho h$. Equation (8) holds also for the case of a semi-infinite cylindrical gas enclosure radiating to the center of its base, but in this case $L_0 = D$ and $u_0 = (C_1/C_3)\rho D/2$. The results obtained from equations (7) and (8) provide smooth continuous transitions between the well-known linear and logarithmic absorption limits as shown in Fig. 2, in which the important information regarding where this transition occurs is clearly indicated. Indeed, the present results represent the very first demonstration of such a transition.

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